

Electromagnetic Structure of the ρ Meson in the Light-Front Quark Model

Ho-Meoyng Choi^a and Chueng-Ryong Ji^b

^a Department of Physics, Kyungpook National University, Daegu, 702-701 Korea

^b Department of Physics, North Carolina State University, Raleigh, NC 27695-8202

We investigate the elastic form factors of the ρ meson in the light-front quark model(LFQM). With the phenomenologically accessible meson vertices including the one obtained by the Melosh transformation frequently used in the LFQM, we find that only the helicity $0 \rightarrow 0$ matrix element of the plus current receives the zero-mode contribution. We quantify the zero-mode contribution in the helicity $0 \rightarrow 0$ amplitude using the angular condition of spin-1 system. After taking care of the zero-mode issue, we obtain the magnetic(μ) and quadrupole(Q) moments of the ρ meson as $\mu = 1.92$ and $Q = 0.43$, respectively, in the LFQM consistent with the Melosh transformation and compare our results with other available theoretical predictions.

I. INTRODUCTION

In the past few years, there has been much theoretical effort [1, 2, 3, 4, 5, 6, 7, 8, 9] to study the so called “zero-mode” [10, 11, 12, 13, 14, 15] contribution to the elastic and weak form factors for spin-0 and 1 systems in light-front dynamics(LFD). The zero-mode is closely related to the off-diagonal elements in the Fock state expansion of the current matrix. In particular, the zero-mode contribution to the form factor $F(q^2)$ can be characterized by the nonvanishing contribution from the off-diagonal elements in the $q^+ \rightarrow 0$ limit, where q^+ is the longitudinal component of the momentum transfer q ($q^\pm = q^0 \pm q^3$, $q^2 = q^+q^- - \mathbf{q}_\perp^2$). In the absence of zero-mode, however, the hadron form factors can be expressed as the convolution of the initial and final LF wave functions; namely, the physical result can be obtained by taking into account only the valence contribution or the diagonal elements in the Fock state expansion.

In this work, we analyze the zero-mode contribution to the elastic form factors of the ρ meson using the LF constituent quark model (or LFQM in short) [16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26], which has been quite successful in predicting various electroweak properties of light and heavy mesons compared to the available data. In our previous LFD analysis [6] of spin-1 electromagnetic form factors, we have shown that the zero-mode complication can exist even in the matrix element of the plus component of the current. Using a covariant model of spin-1 system with the polarization vectors obtained from the LF gauge($\epsilon_\pm^\pm = 0$), we found that only the helicity zero-to-zero amplitude, i.e. $(h', h) = (0, 0)$, can be contaminated by the zero-mode contribution. However, our conclusion in [6] was based on the use of a rather simple vector meson vertex $\Gamma^\mu = \gamma^\mu$. The aim of the present work is to explore our findings in the more phenomenologically accessible ρ meson vertex(see Eq. (13)). This includes the case of the ρ meson vertex obtained by the Melosh transformation frequently used in the LFQM.

Recently, Jaus [3, 8] proposed a covariant light-front approach that develops a way of including the zero-mode contribution and removing spurious form factors proportional to the lightlike vector $\omega^\mu = (1, 0, 0, -1)$. Using the

LFQM vector meson vertex that we include in this work, the author in [8] concluded the existence of a zero-mode in the form factor $F_2(Q^2)$ of a spin-1 meson. Although his calculation was performed in a covariant way not using the helicity components, the form factor F_2 is related with the matrix element of $(h', h) = (+, 0)$ -component of the current(see Eq. (2.6) in [8]) as well as $(h', h) = (0, 0)$ component. Thus, his result indicates that both helicity $(+, 0)$ and $(0, 0)$ amplitudes receive the zero-mode contribution. In the present work, we do not agree¹ with this result but find the zero-mode contribution only in the helicity $(h', h) = (0, 0)$ amplitude. This result is quite significant in the LFQM phenomenology because the absence of the zero-mode contamination in the helicity $(+, 0)$ amplitude can give a tremendous benefit in making reliable predictions on the spin-1 observables as we present in the example of the ρ meson. Our work should be intrinsically distinguished from the formulation involving ω , since our formulation involves neither ω nor any unphysical form factor.

The paper is organized as follows. In Sec. II, we present the Lorentz-invariant electromagnetic form factors and the kinematics for the reference frames used in our analysis. We also discuss the LF helicity basis, the angular condition for spin-1 systems, and the two particular prescriptions used in extracting the physical form factors. In Sec. III, we present our LF calculation varying the vector meson vertex. We employ both the manifestly covariant model vertex(beyond the simple model of $\Gamma^\mu = \gamma^\mu$) and the LFQM vertex consistent with the Melosh transformation. We discuss the LF valence and nonvalence contributions using a plus component of the current and show that only the helicity zero-to-zero am-

¹ We encountered previously a similar situation of disagreement in the zero-mode contribution to the A_1 (or f) form factor for the transition between pseudoscalar and vector mesons. However, after the completion of the work [9], the author of Ref. [8] informed us in a private communication that he completely agrees with our results [9] due to the identity Eq. (3.33) of Ref. [8]. Nonetheless, the communication on the spin-1 elastic form factors presented in this work has not yet been made.

plitude receives zero-mode contribution for the employed meson vertices. In Sec. IV, we present our numerical results for the physical quantities of the ρ meson using the LFQM and compare our results with other available theoretical predictions. Conclusions follow in Sec. V. In Appendix A, we summarize our results of the trace calculations for various helicity components used in our form factor calculations.

II. ELECTROMAGNETIC STRUCTURE OF SPIN-1 SYSTEM

The Lorentz-invariant electromagnetic form factors $F_i (i = 1, 2, 3)$ for a spin-1 particle are defined [27] by the matrix elements of the current J^μ between the initial $|P, h\rangle$ (momentum P and helicity h) and the final $|P', h'\rangle$ eigenstates as follows:

$$\begin{aligned} \langle P', h' | J^\mu | P, h \rangle = & -\epsilon_{h'}^* \cdot \epsilon_h (P + P')^\mu F_1(Q^2) \\ & + (\epsilon_h^\mu q \cdot \epsilon_{h'}^* - \epsilon_{h'}^{\mu*} q \cdot \epsilon_h) F_2(Q^2) \\ & + \frac{(\epsilon_{h'}^* \cdot q)(\epsilon_h \cdot q)}{2M_v^2} (P + P')^\mu F_3(Q^2), \end{aligned} \quad (1)$$

where $q = P' - P$ and $\epsilon_h[\epsilon_{h'}]$ is the polarization vector of the initial[final] meson with the physical mass M_v .

We analyze the matrix elements in the Breit frame($q^+ = 0, q_y = 0, q_x = Q$, and $\mathbf{P}_\perp = -\mathbf{P}'_\perp$) defined by [18, 20, 28, 29, 30, 31]

$$\begin{aligned} q^\mu &= (0, 0, Q, 0) \\ P^\mu &= (M_v\sqrt{1+\kappa}, M_v\sqrt{1+\kappa}, -Q/2, 0) \\ P'^\mu &= (M_v\sqrt{1+\kappa}, M_v\sqrt{1+\kappa}, Q/2, 0), \end{aligned} \quad (2)$$

where $\kappa = Q^2/4M_v^2$ is the kinematic factor. Here, we use the notation $p = (p^+, p^-, p^1, p^2)$ and the metric convention $p^2 = p^+p^- - \mathbf{p}_\perp^2$ with $p^\pm = p^0 \pm p^3$.

Following Bjorken-Drell convention [32], we work with the circular polarization and spin projection $h = \pm = \uparrow\downarrow$. With $\epsilon^\pm(p, \pm) = 0$ from the LF gauge [1], the polarization vector is given by

$$\epsilon^\mu(p, \pm) = \left(0, \frac{2\vec{\epsilon}_\perp(\pm) \cdot \vec{p}_\perp}{p^+}, \vec{\epsilon}_\perp(\pm) \right), \quad (3)$$

which satisfies $p \cdot \epsilon(\pm) = 0$ with $\vec{\epsilon}_\perp(\pm) = \mp 1/\sqrt{2}(1, \pm i)$. Effectively, the initial and final transverse($h = \pm$) and longitudinal($h = 0$) polarization vectors in the Breit frame are given by

$$\begin{aligned} \epsilon^\mu(P, \pm) &= \frac{\mp 1}{\sqrt{2}} \left(0, \frac{-Q}{P^+}, 1, \pm i \right), \\ \epsilon^\mu(P, 0) &= \frac{1}{M_v} \left(P^+, \frac{-M_v^2 + Q^2/4}{P^+}, \frac{-Q}{2}, 0 \right), \\ \epsilon^\mu(P', \pm) &= \frac{\mp 1}{\sqrt{2}} \left(0, \frac{Q}{P^+}, 1, \pm i \right), \\ \epsilon^\mu(P', 0) &= \frac{1}{M_v} \left(P^+, \frac{-M_v^2 + Q^2/4}{P^+}, \frac{Q}{2}, 0 \right). \end{aligned} \quad (4)$$

The covariant form factors of a spin-1 hadron in Eq. (1) can be determined using only the plus component of the current, $I_{h'h}^+(0) \equiv \langle P', h' | J^\mu | P, h \rangle$. As one can see from Eq. (1), while there are only three independent(invariant) form factors, nine elements of $I_{h'h}^+(0)$ can be assigned to the current operator. However, since the current matrix elements $I_{h'h}^+(0)$ must be constrained by the invariance under the LF parity and the LF time-reversal, one can reduce the independent matrix elements of the current down to four, e.g. $I_{++}^+, I_{+-}^+, I_{+0}^+$ and I_{00}^+ [18, 20, 28, 29, 30]. The angular momentum conservation requires an additional constraint on the current operator, which yields the so called ‘‘angular condition’’ $\Delta(Q^2)$ given by [28]

$$\Delta(Q^2) = (1 + 2\kappa)I_{++}^+ + I_{+-}^+ - \sqrt{8\kappa}I_{+0}^+ - I_{00}^+ = 0. \quad (5)$$

Because of the angular condition, only three helicity amplitudes are independent as expected from the three physical form factors. However, the relations between the physical form factors and the matrix elements $I_{h'h}^+$ are not uniquely determined because the number of helicity amplitudes is larger than that of physical form factors. So one may choose which matrix elements to use to extract the form factors. For example, Grach and Kondratyuk(GK) [28] used only $(h', h) = (+, +), (+, -)$ and $(+, 0)$, but not the pure longitudinal $(0, 0)$ component. On the other hand, Brodsky and Hiller(BH) [30] used $(0, 0), (+, 0)$, and $(+, -)$ amplitudes considering that the $(0, 0)$ component gives the most dominant contribution in the high momentum perturbative QCD(pQCD) region. Chung, Coester, Keister and Polyzou(CCKP) [29] involved all helicity states, i.e. $(+, +), (0, 0), (+, 0)$, and $(+, -)$. Among these various choices, we present GK and BH prescriptions for the comparison purpose in this work. In practical computation, instead of calculating the Lorentz-invariant form factors $F_i(Q^2)$, the physical charge(G_C), magnetic(G_M), and quadrupole(G_Q) form factors are often used. The relation between F 's and G 's are given by

$$\begin{aligned} G_C &= F_1 + \frac{2}{3}\kappa G_Q, \\ G_M &= -F_2, \\ G_Q &= F_1 + F_2 + (1 + \kappa)F_3. \end{aligned} \quad (6)$$

The physical form factors(G_C, G_M , and G_Q) in terms of the matrix elements $I_{h'h}^+$ for GK and BH prescriptions are given by

$$\begin{aligned} G_C^{\text{GK}} &= \frac{1}{2P^+} \left[\frac{(3-2\kappa)}{3} I_{++}^+ + \frac{4\kappa}{3} \frac{I_{+0}^+}{\sqrt{2\kappa}} + \frac{1}{3} I_{+-}^+ \right], \\ G_M^{\text{GK}} &= \frac{2}{2P^+} \left[I_{++}^+ - \frac{1}{\sqrt{2\kappa}} I_{+0}^+ \right], \\ G_Q^{\text{GK}} &= \frac{1}{2P^+} \left[-I_{++}^+ + 2 \frac{I_{+0}^+}{\sqrt{2\kappa}} - \frac{I_{+-}^+}{\kappa} \right], \end{aligned} \quad (7)$$

and

$$\begin{aligned}
G_C^{\text{BH}} &= \frac{1}{2P^+(1+2\kappa)} \left[\frac{3-2\kappa}{3} I_{00}^+ + \frac{16}{3} \kappa \frac{I_{+0}^+}{\sqrt{2\kappa}} \right. \\
&\quad \left. + \frac{2}{3} (2\kappa-1) I_{+-}^+ \right], \\
G_M^{\text{BH}} &= \frac{2}{2P^+(1+2\kappa)} \left[I_{00}^+ + \frac{(2\kappa-1)}{\sqrt{2\kappa}} I_{+0}^+ - I_{+-}^+ \right], \\
G_Q^{\text{BH}} &= \frac{-1}{2P^+(1+2\kappa)} \left[I_{00}^+ - 2 \frac{I_{+0}^+}{\sqrt{2\kappa}} + \frac{1+\kappa}{\kappa} I_{+-}^+ \right]. \quad (8)
\end{aligned}$$

Of course, the matrix elements must fulfill the angular

condition given by Eq. (5) for the physical form factors to be independent from the prescriptions (GK or BH).

At zero momentum transfer, these form factors are proportional to the usual static quantities of charge e , magnetic moment μ , and quadrupole moment Q [18, 31]:

$$eG_C(0) = e, \quad eG_M(0) = 2M_v\mu, \quad eG_Q(0) = M_v^2 Q. \quad (9)$$

In principle, these physical form factors are also related to the structure functions $A(Q^2)$, $B(Q^2)$ and the tensor polarization T_{20} [30]:

$$\begin{aligned}
A(Q^2) &= G_C^2 + \frac{2}{3} \kappa G_M^2 + \frac{8}{9} \kappa^2 G_Q^2, \quad B(Q^2) = \frac{4}{3} \kappa (1 + \kappa) G_M^2, \\
T_{20}(Q^2, \theta) &= -\kappa \frac{\sqrt{2}}{3} \frac{\frac{4}{3} \kappa G_Q^2 + 4G_Q G_C + [1/2 + (1 + \kappa) \tan^2(\theta/2)] G_M^2}{A + B \tan^2(\theta/2)}. \quad (10)
\end{aligned}$$

III. MODEL DESCRIPTION AND CALCULATION

As we have shown in our previous work [6], the form factors of a spin-1 particle can be derived from the covariant Bethe-Salpeter (BS) model of $(3+1)$ -dimensional fermion field theory.

The covariant diagram shown in Fig. 1(a) is in general equivalent to the sum of LF valence diagram (b) and the nonvalence diagram (c), where $\alpha = P'^+/P^+ = 1 + q^+/P^+$. The electromagnetic (EM) current $I_{h'h}^\mu(0)$ of a spin-1 particle with equal constituent ($m = m_q = m_{\bar{q}}$) obtained from the covariant diagram of Fig. 1(a) is given by

$$I_{h'h}^\mu(0) = iN_c g^2 \int \frac{d^4 k}{(2\pi)^4} \frac{S_\Lambda(k-P) S_{h'h}^\mu S_\Lambda(k-P')}{[(k-P)^2 - m^2 + i\varepsilon][k^2 - m^2 + i\varepsilon][(k-P')^2 - m^2 + i\varepsilon]}, \quad (11)$$

where N_c is the number of colors and g is the normalization constant modulo the charge factor e_q which can be fixed by requiring the charge form factor to be unity at $q^2 = 0$. In Ref. [6], we replace the point photon-vertex γ^μ by a nonlocal smeared photon-vertex $S_\Lambda(P)\gamma^\mu S_\Lambda(P')$ to regularize the covariant fermion triangle loop in $(3+1)$ dimensions, where $S_\Lambda(P) = \Lambda^2/(P^2 - \Lambda^2 + i\varepsilon)$ and Λ plays the role of a momentum cutoff similar to the Pauli-Villars regularization.

The trace term $S_{h'h}^\mu$ of the quark propagators in Eq. (11) is given by

$$S_{h'h}^\mu = (\epsilon_{h'}^*)_\alpha \text{Tr}[\Gamma^\alpha(\not{p}' + m)\gamma^\mu(\not{p} + m)\Gamma^\beta(\not{k} + m)](\epsilon_h)_\beta, \quad (12)$$

where $p = k - P$, $p' = k - P'$ and $\Gamma^\beta(\Gamma^\alpha)$ is the spinor structure of initial (final) state meson-quark vertex. In our previous work [6], using a rather simple meson vertex, $\Gamma^\mu = \gamma^\mu$, we showed that only S_{00}^+ receives zero-mode

contributions.

The purpose of the present work is to go beyond the simple vertex, $\Gamma^\mu = \gamma^\mu$, and analyze the zero-mode contribution to see if the same conclusion (i.e. zero-mode only in S_{00}^+) can be drawn. For this purpose, we extend the meson vertex to the more general one, which has been used for the phenomenology:

$$\Gamma^\beta = \gamma^\beta - \frac{(2k-P)^\beta}{D}, \quad \Gamma^\alpha = \gamma^\alpha - \frac{(2k-P')^\alpha}{D'}. \quad (13)$$

While in the manifestly covariant model, D and D' may

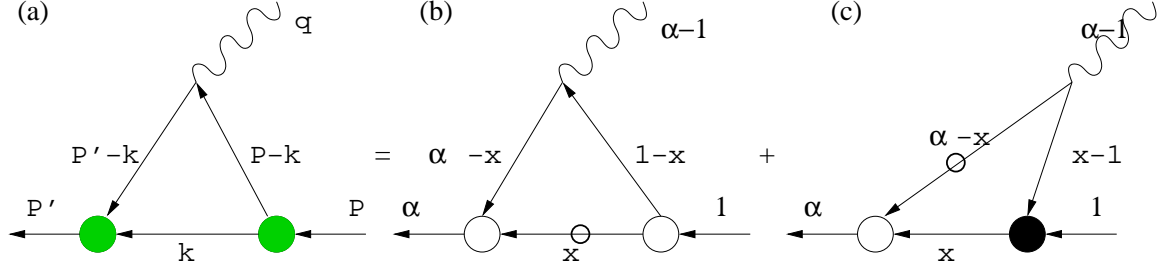


FIG. 1: Covariant triangle diagram (a) is represented as the sum of LF valence diagram (b) defined in $0 < k^+ < P^+$ region and the nonvalence diagram (c) defined in $P^+ < k^+ < P'^+$ region, where $\alpha = P'^+/P^+ = 1 + q^+/P^+$. The large white and black blobs at the meson-quark vertices in (b) and (c) represent the ordinary LF wave function and the nonvalence wave function vertices, respectively. The small white blobs in (b) and (c) represent the (on-shell) mass pole of the quark propagator determined from the k^- -integration.

be written as [33, 34]²

$$\begin{aligned} D_{\text{cov}} &= \frac{2}{M_v} (k \cdot P + M_v m - i\epsilon), \\ D'_{\text{cov}} &= \frac{2}{M_v} (k \cdot P' + M_v m - i\epsilon), \end{aligned} \quad (14)$$

in the standard LQM they are given by [3, 8, 18]

$$\begin{aligned} D_{\text{LQM}} &= \sqrt{\frac{m^2 + \mathbf{k}_{i\perp}^2}{x(1-x)}} + 2m = M_{0i} + 2m, \\ D'_{\text{LQM}} &= \sqrt{\frac{m^2 + \mathbf{k}_{f\perp}^2}{x'(1-x')}} + 2m = M_{0f} + 2m, \end{aligned} \quad (15)$$

where $x' = x/\alpha$, $\alpha = P'^+/P^+ = 1 + q^+/P^+$ (i.e. $x' = x$ in $q^+ = 0$ frame) and

$$\mathbf{k}_{i\perp} = \mathbf{k}_\perp + \frac{x}{2} \mathbf{q}_\perp, \quad \mathbf{k}_{f\perp} = \mathbf{k}_\perp - \frac{x'}{2} \mathbf{q}_\perp. \quad (16)$$

We note that the LF meson vertex (Eq. (13)) with D_{LQM} given by Eq. (15) can be obtained by the standard LF Melosh transformation [16]. The equivalence between $D_{\text{cov}}(D'_{\text{cov}})$ in Eq. (14) and $D_{\text{LQM}}(D'_{\text{LQM}})$ in Eq. (15) can be established only in a very limited case, e.g. zero binding energy limit, $M_v = M_0$.

As we have shown in our previous analysis, with a rather simple but manifestly covariant meson vertex ($\Gamma^\mu = \gamma^\mu$) [6, 9], we can in principle duplicate both the manifestly covariant calculation and the LF calculation to check which form factors are immune to the zero-mode. However, as we have also shown in Refs. [6, 9], we

can directly check the power counting of the longitudinal momentum fraction in the trace term to find out explicitly which helicity amplitude has the zero-mode contribution. No matter which way we check, the results are the same. In this work, we follow the latter procedure to check each helicity amplitude and show explicitly that both phenomenologically accessible meson vertices given by Eqs. (14) and (15) lead to the same conclusion, i.e. the zero-mode contribution exists only in the helicity zero-to-zero amplitude.

Before we proceed, we first summarize our result of the trace term for the '+'-component of the current. Then, we separate the valence contribution from the nonvalence contribution and take the $q^+ \rightarrow 0$ limit of the nonvalence contribution to analyze the zero-mode in each helicity amplitude.

A. Trace Calculation

From Eqs. (12) and (13), we obtain the trace term for the '+'-component of the current as follows

$$\begin{aligned} S_{h'h}^+ &= (\epsilon_{h'}^*)_\alpha \text{Tr}[\Gamma^\alpha(\not{p}' + m)\gamma^+(\not{p} + m)\Gamma^\beta(\not{k} + m)](\epsilon_h)_\beta, \\ &= \text{Tr}[\not{\epsilon}_{h'}^*(\not{p}' + m)\gamma^+(\not{p} + m)\not{\epsilon}_h(\not{k} + m)] \\ &\quad - \frac{\epsilon_h \cdot (2k - P)}{D} \text{Tr}[\not{\epsilon}_{h'}^*(\not{p}' + m)\gamma^+(\not{p} + m)(\not{k} + m)], \\ &\quad - \frac{\epsilon_{h'}^* \cdot (2k - P')}{D'} \text{Tr}[(\not{p}' + m)\gamma^+(\not{p} + m)\not{\epsilon}_h(\not{k} + m)], \\ &\quad + \frac{\epsilon_{h'}^* \cdot (2k - P')}{D'} \frac{\epsilon_h \cdot (2k - P)}{D} \\ &\quad \times \text{Tr}[(\not{p}' + m)\gamma^+(\not{p} + m)(\not{k} + m)], \\ &= S_{1h'h}^+ - S_{2h'h}^+ - S_{3h'h}^+ + S_{4h'h}^+. \end{aligned} \quad (17)$$

Using the following identity

$$\not{p} + m = (\not{p}_{\text{on}} + m) + \frac{1}{2}\gamma^+(p^- - p_{\text{on}}^-), \quad (18)$$

we separate the trace part for each $S_{ih'h}^+(i = 1, 2, 3, 4)$ into the on-mass shell (i.e. $p^2 = m^2$ or $p^- = p_{\text{on}}^- = (m^2 +$

² In a brief presentation of Ref. [34], the authors indicated that they investigated the rho meson form factors using Eq. (14). However, the detailed explicit calculations have not yet been presented. Moreover, the result shown in Ref. [34] was based on the instant-form polarization rather than the light-front polarization so that the direct analysis of light-front helicity amplitudes as discussed in this work has not yet been presented.

$\mathbf{p}_\perp^2)/p^+$ propagating part, $(S_{ih'h}^+)_{\text{on}}$, and the off-shell part, $(S_{ih'h}^+)_{\text{off}}$, i.e. $S_{ih'h}^+ = (S_{ih'h}^+)_{\text{on}} + (S_{ih'h}^+)_{\text{off}}$.

The on-mass shell parts $(S_{ih'h}^+)_{\text{on}}$ are given by

$$\begin{aligned} (S_{1h'h}^+)_{\text{on}} &= \text{Tr}[\not{\epsilon}_{h'}^*(\not{p}'_{\text{on}} + m)\gamma^+(\not{p}_{\text{on}} + m)\not{\epsilon}_h(\not{k}_{\text{on}} + m)], \\ (S_{2h'h}^+)_{\text{on}} &= \frac{\epsilon_h \cdot (2k_{\text{on}} - P)}{D} \\ &\quad \times \text{Tr}[\not{\epsilon}_{h'}^*(\not{p}'_{\text{on}} + m)\gamma^+(\not{p}_{\text{on}} + m)(\not{k}_{\text{on}} + m)], \\ (S_{3h'h}^+)_{\text{on}} &= \frac{\epsilon_{h'}^* \cdot (2k_{\text{on}} - P')}{D'} \\ &\quad \times \text{Tr}[(\not{p}'_{\text{on}} + m)\gamma^+(\not{p}_{\text{on}} + m)\not{\epsilon}_h(\not{k}_{\text{on}} + m)], \\ (S_{4h'h}^+)_{\text{on}} &= \frac{\epsilon_{h'}^* \cdot (2k_{\text{on}} - P')}{D'} \frac{\epsilon_h \cdot (2k_{\text{on}} - P)}{D} \\ &\quad \times \text{Tr}[(\not{p}'_{\text{on}} + m)\gamma^+(\not{p}_{\text{on}} + m)(\not{k}_{\text{on}} + m)], \end{aligned} \quad (19)$$

and the off-shell parts $(S_{ih'h}^+)_{\text{off}}$ are given by

$$\begin{aligned} (S_{1h'h}^+)_{\text{off}} &= \frac{(k^- - k_{\text{on}}^-)}{2} \\ &\quad \times \text{Tr}[\not{\epsilon}_{h'}^*(\not{p}'_{\text{on}} + m)\gamma^+(\not{p}_{\text{on}} + m)\not{\epsilon}_h\gamma^+], \\ (S_{2h'h}^+)_{\text{off}} &= \frac{\epsilon_h^+(k^- - k_{\text{on}}^-)}{D} \\ &\quad \times \text{Tr}[\not{\epsilon}_{h'}^*(\not{p}'_{\text{on}} + m)\gamma^+(\not{p}_{\text{on}} + m)(\not{k}_{\text{on}} + m)] \\ &\quad + \frac{(k^- - k_{\text{on}}^-)}{2D} \epsilon_h \cdot (2k - P) \\ &\quad \times \text{Tr}[\not{\epsilon}_{h'}^*(\not{p}'_{\text{on}} + m)\gamma^+(\not{p}_{\text{on}} + m)\gamma^+], \\ (S_{3h'h}^+)_{\text{off}} &= \frac{\epsilon_{h'}^{*+}(k^- - k_{\text{on}}^-)}{D'} \\ &\quad \times \text{Tr}[(\not{p}'_{\text{on}} + m)\gamma^+(\not{p}_{\text{on}} + m)\not{\epsilon}_h(\not{k}_{\text{on}} + m)] \\ &\quad + \frac{(k^- - k_{\text{on}}^-)}{2D'} \epsilon_{h'}^* \cdot (2k - P') \\ &\quad \times \text{Tr}[(\not{p}'_{\text{on}} + m)\gamma^+(\not{p}_{\text{on}} + m)\not{\epsilon}_h\gamma^+], \\ (S_{4h'h}^+)_{\text{off}} &= \frac{(k^- - k_{\text{on}}^-)}{DD'} [\epsilon_h^+ \epsilon_{h'}^* \cdot (2k_{\text{on}} - P') \\ &\quad + \epsilon_{h'}^{*+} \epsilon_h \cdot (2k_{\text{on}} - P) + \epsilon_h^+ \epsilon_{h'}^{*+}(k^- - k_{\text{on}}^-)] \\ &\quad \times \text{Tr}[(\not{p}'_{\text{on}} + m)\gamma^+(\not{p}_{\text{on}} + m)(\not{k}_{\text{on}} + m)] \\ &\quad + \frac{(k^- - k_{\text{on}}^-)}{2DD'} \epsilon_{h'}^* \cdot (2k - P') \epsilon_h \cdot (2k - P) \\ &\quad \times \text{Tr}[(\not{p}'_{\text{on}} + m)\gamma^+(\not{p}_{\text{on}} + m)\gamma^+]. \end{aligned} \quad (20)$$

In Appendix A, we present the explicit expressions of $(S_{ih'h}^+)$ in terms of LF variables.

Now, by doing the integration over k^- in Eq. (11), one finds the two LF time-ordered contributions to the residues corresponding to the two poles in k^- , the one coming from the interval (I) $0 < k^+ < P^+$ [see Fig. 1(b),

the “valence contribution”], and the other from (II) $P^+ < k^+ < P'^+$ [see Fig. 1(c), the “nonvalence contribution”].

B. Valence contribution

In the valence region of $0 < k^+ < P^+$ as shown in Fig. 1(b), the residue is at the pole of $k^- = k_{\text{on}}^- = [m^2 + \mathbf{k}_\perp^2 - i\varepsilon]/k^+$ (i.e. spectator quark), which is placed in the lower half of complex- k^- plane. Thus, the Cauchy integration over k^- of the plus current, $I_{h'h}^+(0)$ (See Eq. (11)), in the Breit frame given by Eq. (2) yields

$$\begin{aligned} I_{h'h}^{+\text{val}} &= \frac{N_c}{16\pi^3} \int_0^1 \frac{dx}{x(1-x)^2} \\ &\quad \times \int d^2\mathbf{k}_\perp \chi_i(x, \mathbf{k}_{i\perp}) S_{h'h}^+ \chi_f(x, \mathbf{k}_{f\perp}), \end{aligned} \quad (21)$$

where

$$\chi_i(x, \mathbf{k}_{i\perp}) = \frac{g\Lambda^2}{(1-x)(M_v^2 - M_{0i}^2)(M_v^2 - M_{0\Lambda i}^2)} \quad (22)$$

corresponds to the initial state meson vertex function with

$$M_{0\Lambda i}^2 = \frac{m^2 + \mathbf{k}_{i\perp}^2}{x} + \frac{\Lambda^2 + \mathbf{k}_{i\perp}^2}{1-x}. \quad (23)$$

The final state denoted by subscript (f) can be obtained by replacing $(x, \mathbf{k}_{i\perp})$ with $(x', \mathbf{k}_{f\perp})$ in Eqs. (22) and (23). Since $k^- = k_{\text{on}}^-$ in the valence region, $(S_{h'h}^+)_{\text{off}} = 0$ and $S_{h'h}^+ = (S_{h'h}^+)_{\text{on}}$.

C. Nonvalence contribution and zero-mode in $q^+ \rightarrow 0$ limit

In the region $P^+ < k^+ < P'^+ (= P^+ + q^+)$ as shown in Fig. 1(c), the poles are at $k^- \equiv k_m^- = P_1^- + [m^2 + (\mathbf{k}_\perp - \mathbf{P}'_\perp)^2 - i\varepsilon]/(k^+ - P'^+)$ (from the struck quark propagator) and $k^- \equiv k_\Lambda^- = P'^- + [\Lambda^2 + (\mathbf{k}_\perp^2 - \mathbf{P}'_\perp)^2 - i\varepsilon]/(k^+ - P'^+)$ (from the smeared quark-photon vertex $S_\Lambda(k - P')$). Both of them are located in the upper half of the complex k^- plane.

When we do the Cauchy integration over k^- to obtain the LF time-ordered diagrams, we decompose the product of five energy denominators in Eq. (11) into a sum of terms with three energy denominators (see Eq. (21) in Ref.[6]) to avoid the complexity of treating double k^- poles and obtain

$$I_{h'h}^{+nv} = -\frac{g^2 \Lambda^4}{16\pi^3 (\Lambda^2 - m^2)^2} \int_1^\alpha \frac{dx}{xx''(x - \alpha)} \int d^2 \mathbf{k}_\perp \left\{ \frac{S_{h'h}^+(k^- = k_\Lambda^-)}{(M_v^2 - M_{0\Lambda f}^2)(q^2 - M_{\Lambda\Lambda}^2)} - \frac{S_{h'h}^+(k^- = k_\Lambda^-)}{(M_v^2 - M_{0\Lambda f}^2)(q^2 - M_{\Lambda m}^2)} \right. \\ \left. + \frac{S_{h'h}^+(k^- = k_m^-)}{(M_v^2 - M_{0f}^2)(q^2 - M_{mm}^2)} - \frac{S_{h'h}^+(k^- = k_m^-)}{(M_v^2 - M_{0f}^2)(q^2 - M_{m\Lambda}^2)} \right\}, \quad (24)$$

where

$$M_{\Lambda\Lambda}^2 = \frac{\mathbf{k}_\perp'^2 + \Lambda^2}{x''(1 - x'')}, \quad M_{mm}^2 = \frac{\mathbf{k}_\perp'^2 + m^2}{x''(1 - x'')}, \\ M_{\Lambda m}^2 = \frac{\mathbf{k}_\perp'^2 + \Lambda^2}{x''} + \frac{\mathbf{k}_\perp'^2 + m^2}{1 - x''}, \\ M_{m\Lambda}^2 = \frac{\mathbf{k}_\perp'^2 + m^2}{x''} + \frac{\mathbf{k}_\perp'^2 + \Lambda^2}{1 - x''}, \quad (25)$$

with the variables defined by

$$x'' = \frac{1 - x}{1 - \alpha}, \quad \mathbf{k}_\perp'' = \mathbf{k}_\perp + \left(\frac{1}{2} - x'' \right) \mathbf{q}_\perp. \quad (26)$$

Now, the zero-mode [6] appears if the nonvalence diagram does not vanish in $q^+ \rightarrow 0$ limit, i.e.

$$\lim_{q^+ \rightarrow 0} \int_{P^+}^{P^+ + q^+} dk^+(\cdots) \equiv \lim_{\alpha \rightarrow 1} \int_1^\alpha dx(\cdots) \neq 0. \quad (27)$$

To check if this is the case, we count the longitudinal momentum fraction factors in Eq. (24), e.g. in the $q^+ \rightarrow 0$ limit, the 1st term in $I_{h'h}^{+nv}$ becomes

$$I_{h'h}^{+z.m.} \sim \lim_{\alpha \rightarrow 1} \int_1^\alpha \frac{dx}{xx''(x - \alpha)} \frac{S_{h'h}^+(k^- = k_\Lambda^-)}{(M_v^2 - M_{0\Lambda f}^2)(q^2 - M_{\Lambda\Lambda}^2)} \\ = \lim_{\alpha \rightarrow 1} \int_1^\alpha dx \frac{(1 - x)}{(1 - \alpha)} [\cdots] S_{h'h}^+(k_\Lambda^-), \quad (28)$$

where the factor $[\cdots]$ corresponds to the part that is regular as $q^+ \rightarrow 0$ (i.e. $\alpha \rightarrow 1$). Thus, the nonvanishing zero-mode contribution is possible only if the longitudinal momentum fraction of $S_{h'h}^+(k^-)$ is proportional to $(1 - x)^{-s}$ with $s \geq 1$. Otherwise, there is no zero-mode contribution since the integration range shrinks to zero as $q^+ \rightarrow 0$. Other three terms in Eq. (24) have the same behavior as the first term shown in Eq. (28).

Since we already know that the helicity zero-to-zero component, S_{00}^+ , gives a nonvanishing zero-mode contribution at the level of simple vertex, $\Gamma^+ = \gamma^+$ [6], we need to focus on other helicity components for the vector meson vertex given by Eq. (13). In the nonvalence region ($P^+ < k^+ < P'^+$), since the LF energy poles, k_{m1}^- and $k_{\Lambda1}^-$, are proportional to $k^- \sim 1/(1 - x)$, we know that the D -terms of covariant and LFQM vertices in Eq. (13) behave as

$$D_{\text{cov}} \sim \frac{1}{1 - x}, \quad D_{\text{LFQM}} \sim \sqrt{\frac{1}{1 - x}}, \quad (29)$$

by counting only the singular longitudinal momenta in the $\alpha \rightarrow 1$ (i.e. $x \rightarrow 1$) limit. We also note that the invariant mass in the D_{LFQM} for the initial state vector meson has to be replaced by

$$M_{0i}^2 = \frac{m^2 + \mathbf{k}_{i\perp}^2}{x} + \frac{m^2 + \mathbf{k}_{i\perp}^2}{1 - x} \\ \rightarrow \frac{m^2 + \mathbf{k}_{i\perp}^2}{x} + \frac{m^2 + \mathbf{k}_{i\perp}^2}{x - 1} \quad (30)$$

in the nonvalence region ($x > 1$) since the initial state vertex becomes the non-wavefunction vertex [4] (see Fig. 1(c)). Nevertheless, the power of the singular term in $1 - x$ given in Eq. (29) remains the same. Now, from the explicit forms of the trace terms $S_{ih'h}^+$ given in Appendix A, we could determine the existence/non-existence of the zero-mode for each helicity component. Regardless of using covariant or LFQM vertices, we find that there are no singular terms in $1/(1 - x)$ for the helicity $(++, +-, +0)$ components. The most problematic term regarding on the zero-mode was found to be the 2nd term $(S_{2+0}^+)_{\text{off}}$ given by Eq. (A6), which is proportional to $(k^- - k_{\text{on}}^-)/D$. While $(k^- - k_{\text{on}}^-)/D_{\text{cov.}}$ is regular in $1/(1 - x)$, $(k^- - k_{\text{on}}^-)/D_{\text{LFQM}} \sim \sqrt{1/(1 - x)}$, i.e. S_{+0}^+ itself shows singular behavior in $\alpha \rightarrow 1$ limit for the LFQM vertex. However, from Eq. (28), the net result of the zero-mode contribution to I_{+0}^{+nv} for $D = D_{\text{LFQM}}$ is shown to be zero because

$$I_{+0}^{+z.m.} \sim \lim_{\alpha \rightarrow 1} \int_\alpha^1 dx \frac{(1 - x)}{(1 - \alpha)} [\cdots] \sqrt{\frac{1}{1 - x}} \\ = \lim_{\alpha \rightarrow 1} \int_0^1 dz \frac{(1 - \alpha)(1 - z)}{\sqrt{(1 - \alpha)(1 - z)}} [\cdots] \\ = 0, \quad (31)$$

where the variable change $x = \alpha + (1 - \alpha)z$ was made and the term $[\cdots]$ again corresponds to the regular part in $q^+ \rightarrow 0$ limit. For helicity (00) component, the zero-mode contribution comes from $(S_{100}^+)_{\text{off}}$ for both $D = D_{\text{cov}}$ and D_{LFQM} (see Eq. (A8)).

Thus, as in the case of Dirac coupling $\Gamma^\mu = \gamma^\mu$, we find that only I_{00}^+ receives the zero-mode contribution for the more general vertex structures given by Eq. (13) with $D_{\text{cov.}}$ and D_{LFQM} . Accordingly, we can compute the electromagnetic structure of the spin-1 particle using only the valence contribution as far as I_{00}^+ is avoided (e.g. GK prescription). Moreover, we may identify the zero-mode contribution to I_{00}^+ using the angular condition given by

Eq. (5), i.e.

$$I_{00}^{+z.m.} = (1 + 2\kappa)I_{++}^{+val} + I_{+-}^{+val} - \sqrt{8\kappa}I_{+0}^{+val} - I_{00}^{+val}, \quad (32)$$

in the $q^+ = 0$ frame with the LF gauge $\epsilon_{\pm}^{\pm} = 0$.

Thus far, we relied on the generalization of the meson vertex (Eq. (13)) while we kept the use of smeared photon-vertex as adopted in Ref. [6]. Although the spin-orbit part of the LFQM can be incorporated by this generalization, the radial part of the LF wavefunction given by Eq. (22) may serve only semi-realistic calculation of physical observables as discussed by others [8, 37]. For the more realistic calculation, one may need to replace the LF radial wavefunction by the one which has much more phenomenological support. For this purpose, we may utilize the LFQM presented in Ref. [22], which has been quite successful in predicting various static properties of low lying hadrons. Comparing χ in Eq. (21) with our light-front wave function given by Ref. [22], we get

$$\chi_i(x, \mathbf{k}_{i\perp}) = \sqrt{\frac{8\pi^3}{N_c}} \sqrt{\frac{\partial k_z}{\partial x} \frac{[x(1-x)]^{1/2}}{M_{0i}}} \phi(x, \mathbf{k}_{i\perp}), \quad (33)$$

where the Jacobian of the variable transformation $\mathbf{k} = (k_z, \mathbf{k}_{\perp}) \rightarrow (x, \mathbf{k}_{\perp})$ is obtained as $\partial k_z / \partial x = M_{0i} / [4x(1-x)]$ and the radial wave function is given by

$$\phi(\mathbf{k}^2) = \sqrt{\frac{1}{\pi^{3/2}\beta^3}} \exp(-\mathbf{k}^2/2\beta^2), \quad (34)$$

which is normalized as

$$\int d^3k |\phi(\mathbf{k}^2)|^2 = \int_0^1 dx \int d^2\mathbf{k}_{\perp} \left(\frac{\partial k_z}{\partial x} \right) |\phi(x, \mathbf{k}_{\perp})|^2 = 1. \quad (35)$$

In the next section, we shall investigate the ρ -meson electromagnetic properties using this model. We should note that this replacement of the radial part of the LF wavefunction cannot alter our conclusion of zero-mode (i.e. only in S_{00}^+) because the use of Eq. (33) is limited just for the valence contribution.

In summary, the LFQM described above provides the calculation of physical form factors with just the valence contribution of $I_{h'h}^+$ except the component of $(h', h) = (0, 0)$; i.e.

$$I_{h'h}^{+LFQM} = \int_0^1 \frac{dx}{2(1-x)} \int d^2\mathbf{k}_{\perp} \sqrt{\frac{\partial k_z}{\partial x}} \sqrt{\frac{\partial k'_z}{\partial x}} \times \phi(x, \mathbf{k}_{i\perp}) \phi^*(x, \mathbf{k}_{f\perp}) \frac{(S_{h'h}^+)_{on}}{M_{0i}M_{0f}}. \quad (36)$$

Thus, in the GK prescription where I_{00}^+ is not used, it is easy to verify $G_C^{GK}(0) = I_{++}^{+LFQM}/2P^+ = 1$ because at $Q = 0$, $M_{0i}^2 = M_{0f}^2 = M_0^2 = (m^2 + \mathbf{k}_{\perp}^2)/x(1-x)$, $(S_{++}^+)_{on} = 4P^+(1-x)M_0^2$ from a straightforward evaluation using Eq. (A1) and $(S_{+-}^+)_{on} = 0$ from Eq. (A3).

However, the BH prescription involves I_{00}^+ component and $G_C^{BH}(0) = I_{00}^{+LFQM}/2P^+ \neq 1$ due to the zero-mode

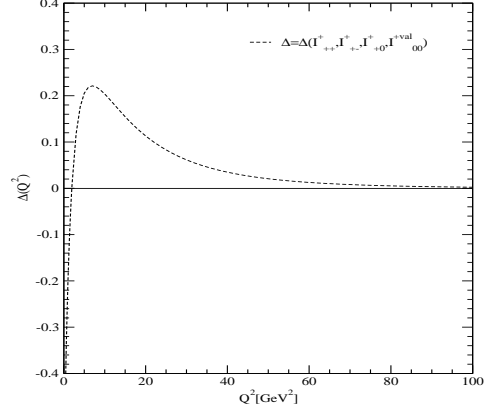


FIG. 2: The angular condition given by Eq. (5) or (32) in the Breit frame, i.e. $\Delta(Q^2) = I_{00}^{+z.m.}$. Note that the zero-mode contribution is necessary to satisfy the angular condition even at $Q^2 = 0$, where $\Delta(0) = -0.65$ in our model calculation.

contribution. It has been shown in Ref. [35] that the additive model for the current operator of interacting constituents is consistent with the angular condition only for the first two terms of the expansion of the plus(good) current in powers of the momentum transfer Q . Indeed, we can show that the zero-mode contribution to I_{00}^{+LFQM} vanishes in the zero-binding limit (i.e. $M_v = M_0$) and the angular condition $\Delta(0) = 0$ in this case. At the same token, we can show that $I_{++}^{+LFQM} = I_{00}^{+LFQM}$ (i.e. $(S_{++}^+)_{on} = (S_{00}^+)_{on}$ from Eqs. (A1) and (A7)) at $Q = 0$ in the zero-binding limit. In most previous LFQM analyses [16, 18, 36], the authors used not only the meson vertex factor given by Eq. (13) together with Eq. (15) but also the zero binding energy prescription, i.e. $M_v = M_0$, which is equivalent to replace M_v in $\epsilon^\mu(P, 0)[\epsilon^\mu(P', 0)]$ with $M_{0i}[M_{0f}]$. In that case, the angular condition is zero at $Q^2 = 0$. However, in this work, we showed an explicit example which gives $\Delta(0) \neq 0$ even if the plus current is used. In Ref. [6], $\Delta(0) \neq 0$ was also shown. Thus, the zero-mode contribution is in general necessary even at $Q^2 = 0$ for the I_{00}^+ amplitude.

IV. NUMERICAL RESULTS

The model parameters used in our analysis are $m = 0.22$ GeV and $\beta = 0.3659$ GeV, which were obtained from the linear confining potential of our QCD-motivated effective Hamiltonian in LFQM [22], as well as $M_v = 0.77$ GeV.

First, we show in Fig. 2 the angular condition $\Delta(Q^2)$ given by Eq. (5) or Eq. (32) in the Breit frame defined by Eq. (2). In this particular reference frame, $\Delta(Q^2)$ is equal to the zero-mode from the helicity zero-to-zero

TABLE I: Magnetic dipole(μ) and quadrupole(Q) moments in units of $e/2M_v$ and e/M_v^2 , respectively.

References	This work	[8]	[46]	[40]
μ	1.92	1.83	2.3	2.0 ± 0.3
Q	0.43	0.33	0.45	-

component, $I_{00}^{+z.m.}$ in Eq. (32). Note that $\Delta(Q^2) \neq 0$ even at $Q^2 = 0$ (See the discussion just before this section.). Unless the binding-energy zero-limit ($M_v = M_0$) is taken, I_{00}^+ is not immune to the zero-mode even at $Q^2 = 0$, but it eventually goes to zero as Q^2 becomes very large.

In Fig. 3, we plot the physical form factors, G_C , G_M , and G_Q , respectively. The solid lines represent the full solutions, i.e. GK prescription in Eq. (7) or equivalently BH prescription in Eq. (8) including the zero-mode contribution to I_{00}^+ , i.e. $I_{00}^{+full} = I_{00}^{+val} + I_{00}^{+z.m.}$, where $I_{00}^{+z.m.}$ is obtained from Eq. (32). As they should be, the full solutions are found to be independent from the choice of prescription. The dashed lines represent the valence contributions to the form factors in the BH prescription. The dashed-dotted lines represent the zero-mode from the helicity zero-to-zero component, which turns out to be exactly the same with the difference between solid and dashed lines. For the charge form factor, $G_C(Q^2)$, we found it has a zero around $Q^2 = 5.5 \text{ GeV}^2$, which is not shown in the figure. Also, the nonvanishing zero-mode contribution to I_{00}^+ at $Q^2 = 0$ is apparent in Fig. 3.

We also obtain the magnetic (in unit of $e/2M_v$) and quadrupole (in unit of e/M_v^2) moments for the ρ meson as

$$\mu = 1.92, \quad Q = 0.43.$$

Our result for the magnetic moment without involving the zero-mode is comparable with $\mu = 1.83$ in Ref. [8] and the one in Ref. [38], in which the author found $\mu < 2$ by considering the low energy limit of the radiative amplitudes in conjunction with the amplitude calculated by the hard-pion technique. The recent light-cone QCD sum rule [39] and the traditional QCD sum rule [40] reported $\mu = 2.3 \pm 0.5$ and 2.0 ± 0.3 , respectively. We note that the author of Ref. [40] also preferred $\mu < 2$. On the other hand, the previous LFQM [18, 36] and the Dyson-Schwinger model [41, 42] predicted $\mu > 2$. We summarize in Table I our results of magnetic dipole(μ) and quadrupole(Q) moments compared with other theoretical predictions.

In Fig. 4, we plot the structure functions $A(Q^2)$, $B(Q^2)$ and their ratio $\log_{10}(B/A)$ up to $Q^2 = 10 \text{ GeV}^2$. The solid and dashed lines represent the full results and the contributions only from the $I_{00}^{+full} (= I_{00}^{+val} + I_{00}^{+z.m.})$ component, respectively. The dominance of helicity zero-to-zero amplitude at high Q^2 region [30, 43, 44] has been discussed in the context of PQCD counting rules and the naturalness condition [45]. However, the analysis of angular condition [43] reveals that the subleading contri-

bution $I_{+0}^+ \sim \frac{\Lambda_{QCD}}{Q} I_{00}^+$ and $I_{++}^+ \sim I_{+-}^+ \sim \left(\frac{\Lambda_{QCD}}{Q}\right)^2 I_{00}^+$ are not as negligible as one might naively expect from PQCD. Our LFQM results indicate that the dominance of I_{00}^+ is realized in $A(Q^2)$ but not in $B(Q^2)$ (or $\log(B/A)$), supporting the significance of the subleading contribution discussed in Ref. [43]. For the magnetic form factor $G_M(Q^2)$ in Eq. (8), both I_{00}^+ and I_{+0}^+ terms contribute at the same order and the I_{+0}^+ contribution at large Q^2 region (even at $Q^2 \sim 100 \text{ GeV}^2$) is not negligible. This explains why we have discrepancies between the full results and I_{00}^+ dominances for the calculations of $B(Q^2)$ and $\log_{10}(B/A)$.

In Fig. 5, we show the full solution of the tensor polarization $T_{20}(Q^2)$ at $\theta = 20^\circ$ (dashed line) and at $\theta = 70^\circ$ (solid line), respectively. For comparison, we also plot the I_{00}^{+full} contribution to $T_{20}(Q^2)$ at $\theta = 20^\circ$ (dot-dashed line) and at $\theta = 70^\circ$ (dotted line), respectively. The $T_{20}(Q^2)$ results also indicate the non-negligible subleading contribution [43] even at a rather large Q^2 region.

V. CONCLUSION

In this work, we investigated the electromagnetic structure of the ρ meson in the light-front quark model. The two vector meson vertices are analyzed, i.e. the manifestly covariant vertex $D = D_{cov.}$ and the LFQM vertex $D = D_{LFQM}$ consistent with the Melosh transformation. Using the power counting method for each helicity amplitude, we found that only the helicity zero-to-zero amplitude receives the zero-mode contribution regardless of $D = D_{cov.}$ or $D = D_{LFQM}$. Other helicity amplitudes such as $(h', h) = (+, +)$, $(+, -)$ and $(+, 0)$ are found to be immune to the zero-mode for both vertices. Our finding is different from that in Ref. [8], in which the author found the nonvanishing zero-mode contribution to the helicity zero-to-plus component as well¹. This is significant because the absence of zero-mode contribution in I_{+0}^+ allows that the pure valence contribution in LFD can give the full result of the physical form factors.

Further, we identified the zero-mode contribution to I_{00}^+ using the angular condition given by Eq. (32). Our result of $\Delta(Q^2)$ exhibits the nonvanishing zero-mode even at $Q^2 = 0$ unless the binding-energy zero limit ($M_v = M_0$) is taken. This does not contradict the findings in the additive model for the current operator of interacting constituents discussed in Ref. [35]. Indeed, our calculation with $D = D_{LFQM}$ for $(h', h) = (+, +)$ and $(+, -)$ are equivalent to the previous LFQM [18, 36] calculations based on the Melosh transformation. However, our calculations involving helicity zero polarization vector, e.g. $(h', h) = (+, 0)$, cannot be the same with the previous Melosh-based calculations [18, 36] unless the zero binding limit $M_v = M_0$ is taken. Thus, it appears important to analyze the difference in the physical observables including the binding energy effects. As we presented in

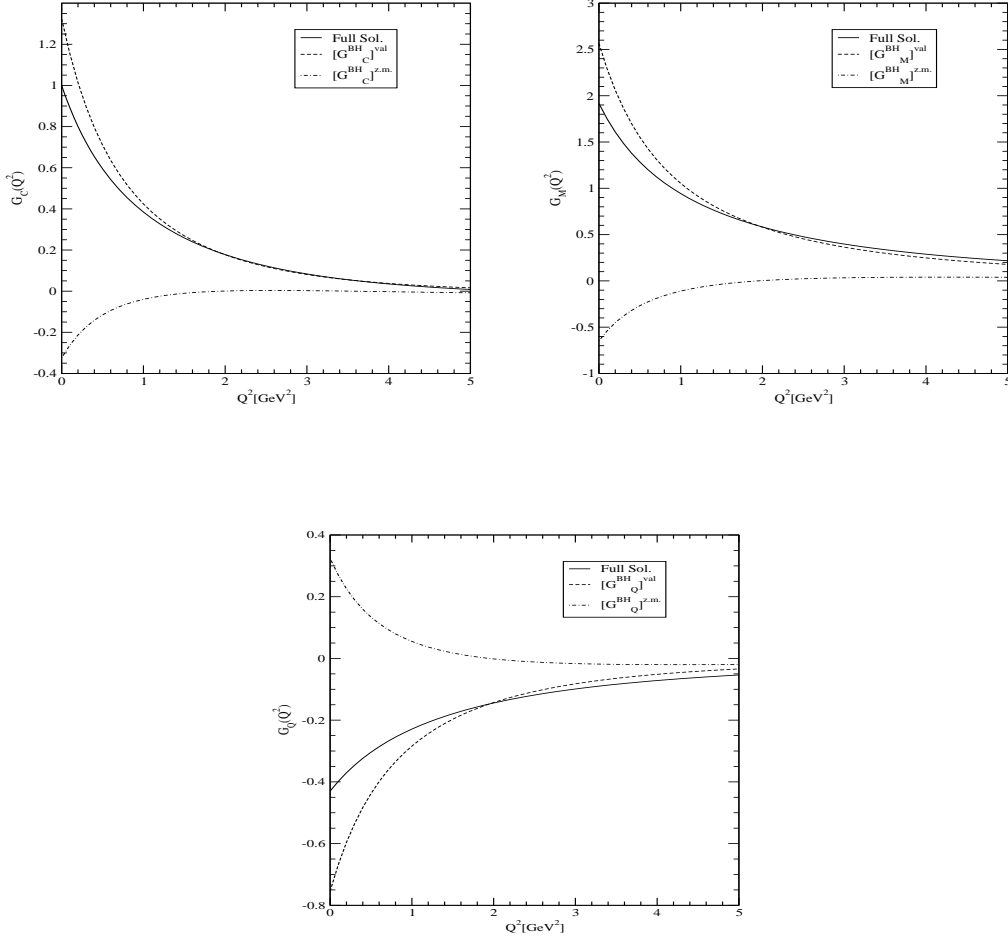


FIG. 3: Form factors of the ρ meson, i.e. charge($G_C(Q^2)$), magnetic($G_M(Q^2)$), and quadrupole($G_Q(Q^2)$) form factors. The solid and dashed lines represent the full(i.e. valence+zero-mode) solution and valence contribution(I_{00}^{+val}) contribution to the form factors, respectively. The dash-dotted line represents the zero-mode contribution, i.e. full solution – valence contribution.

this work, our phenomenological LFQM prediction including the binding energy effect(i.e. $M_v \neq M_0$) leads to $\mu = 1.92 < 2$, which is rather different from the previous results($\mu > 2$) based on the free Melosh transformation [18, 36, 46].

Finally, our numerical results on $B(Q^2)$ and $T_{20}(Q^2)$ at large Q^2 region support the significance of the subleading contribution, e.g. I_{+0}^+ contribution in $B(Q^2)$, found from the analysis of the angular condition in Ref. [43].

Acknowledgments

We thank Ben Bakker for many helpful discussions. This work was supported in part by the grant from

the U.S. Department of Energy (DE-FG02-96ER40947), the National Science Foundation (INT-9906384) and the Korea Research Foundation(KRF-2003-003-C00038). H.M.C would like to thank the staff of the Department of Physics at NCSU for their kind hospitality. The National Energy Research Scientific Computer Center is also acknowledged for the grant of computing time.

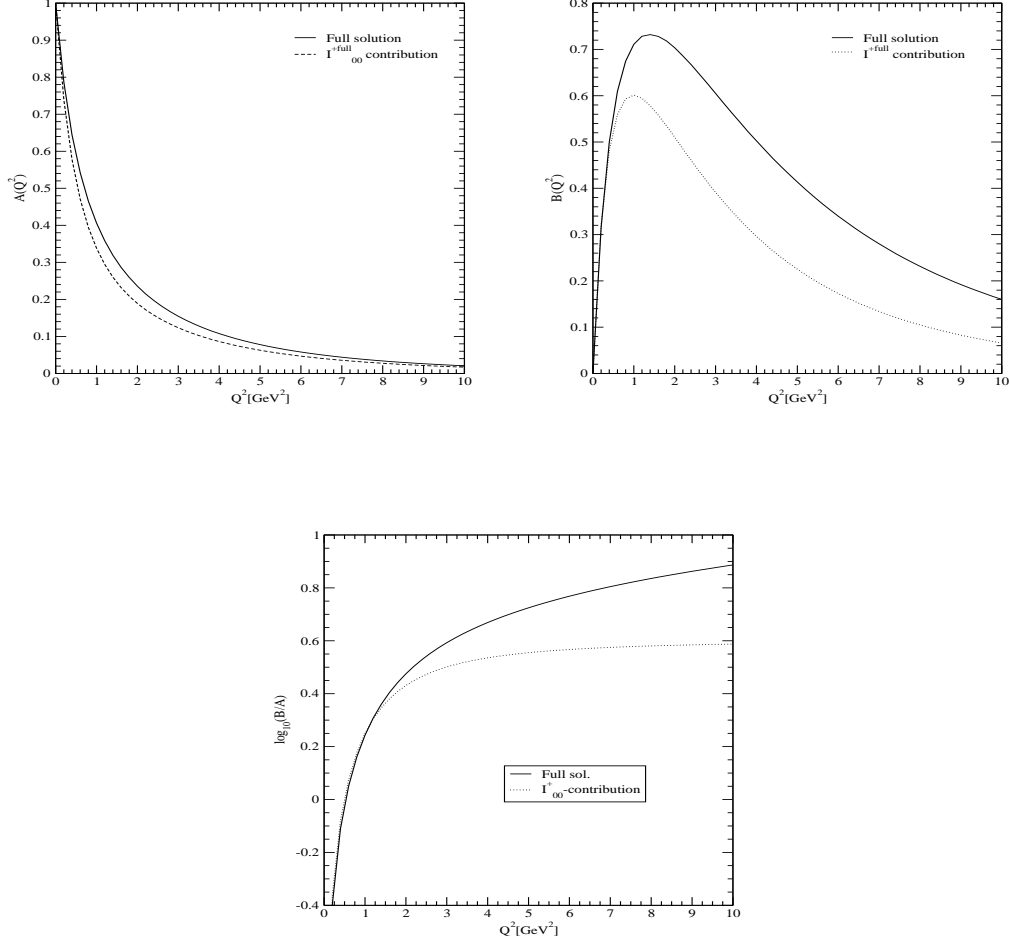


FIG. 4: Structure functions $A(Q^2)$, $B(Q^2)$, and their ratio $\log_{10}(B/A)$. Solid and dotted lines represent the full results and the contributions only from the $I_{00}^{+\text{full}} = I^{+\text{val}} + I^{+\text{z.m.}}$ component.

APPENDIX A: SUMMARY OF TRACE TERMS IN HELICITY AMPLITUDES

In the Breit frame with the LF gauge given by Eqs. (2) and (3), the explicit forms of the trace terms in Eqs. (19) and (20) for each helicity component are given as follows.

(I) helicity $(++)$ -component:

$$\begin{aligned}
 (S_{1++})_{\text{on}} &= \frac{4P^+}{x} \left[m^2 + (2x^2 - 2x + 1) \left(\mathbf{k}_\perp^2 - \frac{x^2}{4} Q^2 - ixk_y Q \right) \right], \\
 (S_{2++})_{\text{on}} &= -2P^+ \left(\frac{m}{D} \right) \left[8(1-x)\mathbf{k}_\perp^2 + x(2x^2 - 2x + 1)Q^2 + 2k_x Q + 2ik_y Q(2x-1)^2 \right], \\
 (S_{3++})_{\text{on}} &= -2P^+ \left(\frac{m}{D'} \right) \left[8(1-x)\mathbf{k}_\perp^2 + x(2x^2 - 2x + 1)Q^2 - 2k_x Q + 2ik_y Q(2x-1)^2 \right], \\
 (S_{4++})_{\text{on}} &= \frac{4P^+}{DD'} \left[\mathbf{k}_\perp^2 - \frac{x^2}{4} Q^2 - ixk_y Q \right] \left[(1-x)(M_{0i}^2 + M_{0f}^2 - 8m^2) - xQ^2 \right]
 \end{aligned} \tag{A1}$$

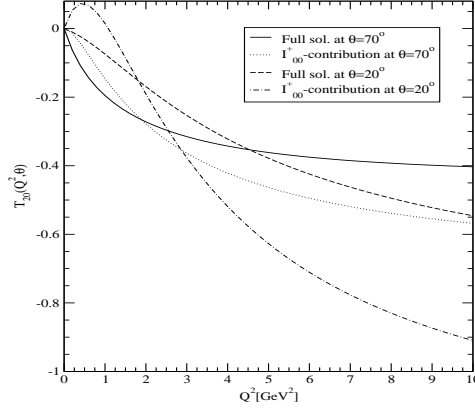


FIG. 5: Tensor polarization $T_{20}(Q^2)$ at $\theta = 70^\circ$ (solid line) and $\theta = 20^\circ$ (dashed line). For comparison, we also show the I_{00}^+ dominance at $\theta = 70^\circ$ (dotted line) and $\theta = 20^\circ$ (dot-dashed line).

and

$$\begin{aligned}
 (S_{1++})_{\text{off}} &= 4(P^+)^2(k^- - k_{\text{on}}^-)(1-x)^2, \\
 (S_{2++})_{\text{off}} &= (S_{3++})_{\text{off}} = 0, \\
 (S_{4++})_{\text{off}} &= \frac{8(P^+)^2}{DD'}(1-x)^2(k^- - k_{\text{on}}^-) \left[\mathbf{k}_\perp^2 - \frac{x^2}{4}Q^2 - ixk_yQ \right].
 \end{aligned} \tag{A2}$$

In the nonvalence region where $k^- \sim 1/(1-x)$, $D = D_{\text{cov.}} \sim 1/(1-x)$ and $D = D_{\text{LFQM}} \sim \sqrt{1/(1-x)}$. Thus, we find from the power counting for the longitudinal momentum fraction that all the off-shell trace terms $(S_{i++})_{\text{off}}$ ($i = 1, 2, 3, 4$) are regular as $q^+ \rightarrow 0$ (or equivalently $x \rightarrow 1$). Therefore, there is no zero-mode in the helicity $(++)$ component.

(II) helicity $(+-)$ -component:

$$\begin{aligned}
 (S_{1+-})_{\text{on}} &= 2P^+(1-x)[4(k^L)^2 - x^2Q^2], \\
 (S_{2+-})_{\text{on}} &= 2P^+\left(\frac{m}{D}\right)(2k^L + xQ)[(2x^2 - 2x + 1)Q + 4(1-x)k^L], \\
 (S_{3+-})_{\text{on}} &= -2P^+\left(\frac{m}{D'}\right)(2k^L - xQ)[(2x^2 - 2x + 1)Q - 4(1-x)k^L], \\
 (S_{4+-})_{\text{on}} &= -\frac{P^+}{DD'}[4(k^L)^2 - x^2Q^2][(1-x)(M_{0i}^2 + M_{0f}^2 - 8m^2) - xQ^2]
 \end{aligned} \tag{A3}$$

and

$$\begin{aligned}
 (S_{1+-})_{\text{off}} &= (S_{2+-})_{\text{off}} = (S_{3+-})_{\text{off}} = 0, \\
 (S_{4+-})_{\text{off}} &= -\frac{2(P^+)^2}{DD'}(k^- - k_{\text{on}}^-)(1-x)^2[4(k^L)^2 - x^2Q^2].
 \end{aligned} \tag{A4}$$

Again, from the power counting for the longitudinal momentum fraction, $(S_{4+-})_{\text{off}}$ is regular as $x \rightarrow 1$. Therefore, there is no zero-mode in the helicity $(+-)$ component.

(III) helicity (+0)-component:

$$\begin{aligned}
(S_{1+0}^+)_{\text{on}} &= \frac{P^+ \sqrt{2}}{M_v} (2k^L - xQ)(2x-1)(1-x)(M_{0i}^2 + M_v^2), \\
(S_{2+0}^+)_{\text{on}} &= -\frac{4P^+}{M_v \sqrt{2}} \left(\frac{m}{D}\right) [(1-x)M_{0i}^2 - xM_v^2][(2x^2 - 2x + 1)Q + 4(1-x)k^L], \\
(S_{3+0}^+)_{\text{on}} &= \frac{4P^+}{M_v \sqrt{2}} \left(\frac{m}{D'}\right) (2x-1)(1-x)(M_{0i}^2 + M_v^2)(2k^L - xQ), \\
(S_{4+0}^+)_{\text{on}} &= \frac{P^+ \sqrt{2}}{M_v DD'} (2k^L - xQ)[(1-x)M_{0i}^2 - xM_v^2][(1-x)(M_{0i}^2 + M_{0f}^2 - 8m^2) - xQ^2], \tag{A5}
\end{aligned}$$

and

$$\begin{aligned}
(S_{1+0}^+)_{\text{off}} &= -\frac{4(P^+)^2}{M_v \sqrt{2}} (k^- - k_{\text{on}}^-)(1-x)(2k^L + xQ), \\
(S_{2+0}^+)_{\text{off}} &= -\frac{4(P^+)^2}{M_v \sqrt{2}} \left(\frac{m}{D}\right) (k^- - k_{\text{on}}^-)[(2x^2 - 2x + 1)Q + 4(1-x)k^L], \\
(S_{3+0}^+)_{\text{off}} &= -\frac{4(P^+)^2}{M_v \sqrt{2}} \left(\frac{m}{D'}\right) (k^- - k_{\text{on}}^-)(1-x)(2k^L - xQ), \\
(S_{4+0}^+)_{\text{off}} &= \frac{(P^+)^2 \sqrt{2}}{M_v DD'} (k^- - k_{\text{on}}^-)(2k^L - xQ)[(1-x)(M_{0i}^2 + M_{0f}^2 - 8m^2) - xQ^2] \\
&\quad + \frac{2\sqrt{2}(P^+)^2}{M_v DD'} (k^- - k_{\text{on}}^-)(1-x)^2(2k^L - xQ)[(1-x)M_{0i}^2 - xM_v^2] \\
&\quad + \frac{2\sqrt{2}(P^+)^3}{M_v DD'} (k^- - k_{\text{on}}^-)^2(1-x)^2(2k^L - xQ). \tag{A6}
\end{aligned}$$

From the power counting of the longitudinal momentum fraction in the nonvalence region, we find only $(S_{2+0}^+)_{\text{off}}$ for $D = D_{\text{LFQM}}$ shows singular behavior, i.e $(S_{2+0}^+)_{\text{off}} \sim \sqrt{1/(1-x)}$ as $x \rightarrow 1$. However, as we show in Eq. (31), the resulting zero-mode contribution vanishes due to the prefactor involved in the integration. Therefore, there is no zero-mode contribution to the helicity (+0) component.

(IV) helicity (00)-component:

$$\begin{aligned}
(S_{100}^+)_{\text{on}} &= \frac{4P^+}{M_v^2} x(1-x)^2(M_{0i}^2 + M_v^2)(M_{0f}^2 + M_v^2), \\
(S_{200}^+)_{\text{on}} &= \frac{4P^+}{M_v^2} \left(\frac{m}{D}\right) (2x-1)(1-x)(M_{0f}^2 + M_v^2)[(1-x)M_{0i}^2 - xM_v^2], \\
(S_{300}^+)_{\text{on}} &= \frac{4P^+}{M_v^2} \left(\frac{m}{D'}\right) (2x-1)(1-x)(M_{0i}^2 + M_v^2)[(1-x)M_{0f}^2 - xM_v^2], \\
(S_{400}^+)_{\text{on}} &= \frac{2P^+}{M_v^2 DD'} [(1-x)M_{0i}^2 - xM_v^2][(1-x)M_{0f}^2 - xM_v^2][(1-x)(M_{0i}^2 + M_{0f}^2 - 8m^2) - xQ^2], \tag{A7}
\end{aligned}$$

and

$$\begin{aligned}
(S_{100}^+)_{\text{off}} &= \frac{4(P^+)^2}{M_v^2} (k^- - k_{\text{on}}^-) [m^2 + \mathbf{k}_\perp^2 - x^2 Q^2/4], \\
(S_{200}^+)_{\text{off}} &= \frac{4(P^+)^2}{M_v^2} \left(\frac{m}{D} \right) (k^- - k_{\text{on}}^-) (1-x)(2x-1)(M_{0f}^2 + M_v^2) \\
&\quad - \frac{4(P^+)^2}{M_v^2} \left(\frac{m}{D} \right) (k^- - k_{\text{on}}^-) (1-x) [(1-x)M_{0i}^2 - xM_v^2 + (k^- - k_{\text{on}}^-)P^+], \\
(S_{300}^+)_{\text{off}} &= \frac{4(P^+)^2}{M_v^2} \left(\frac{m}{D'} \right) (k^- - k_{\text{on}}^-) (1-x)(2x-1)(M_{0i}^2 + M_v^2) \\
&\quad - \frac{4(P^+)^2}{M_v^2} \left(\frac{m}{D'} \right) (k^- - k_{\text{on}}^-) (1-x) [(1-x)M_{0f}^2 - xM_v^2 + (k^- - k_{\text{on}}^-)P^+], \\
(S_{400}^+)_{\text{off}} &= \frac{2(P^+)^2}{M_v^2 DD'} (k^- - k_{\text{on}}^-) [(1-x)(M_{0i}^2 + M_{0f}^2) - 2xM_v^2] [(1-x)(M_{0i}^2 + M_{0f}^2 - 8m^2) - xQ^2] \\
&\quad + \frac{2(P^+)^3}{M_v^2 DD'} (k^- - k_{\text{on}}^-)^2 [(1-x)(M_{0i}^2 + M_{0f}^2 - 8m^2) - xQ^2] \\
&\quad + \frac{4(P^+)^2}{M_v^2 DD'} (k^- - k_{\text{on}}^-) (1-x)^2 \left\{ [(1-x)M_{0i}^2 - xM_v^2] [(1-x)M_{0f}^2 - xM_v^2] \right. \\
&\quad \left. + P^+ (k^- - k_{\text{on}}^-) [(1-x)(M_{0i}^2 + M_{0f}^2) - 2xM_v^2] + (P^+)^2 (k^- - k_{\text{on}}^-)^2 \right\}. \quad (\text{A8})
\end{aligned}$$

From the power counting of the longitudinal momentum fraction, only $(S_{100}^+)_{\text{off}}$ is singular for both $D_{\text{cov.}}$ and D_{LFQM} , i.e. $(S_{100}^+)_{\text{off}} \sim 1/(1-x)$ as $x \rightarrow 1$, which gives the zero-mode contribution to the helicity zero-to-zero component.

-
- | | |
|---|---|
| <p>[1] S. J. Brodsky, H. C. Pauli, and S. S. Pinsky, Phys. Rep. 301, 299 (1998).
 [2] J. Carbonell, B. Desplanques, V.A. Karmanov, and J.-F. Mathiot, Phys. Rep. 300, 215 (1998).
 [3] W. Jaus, Phys. Rev. D 60, 054026 (1999).
 [4] C.-R. Ji and H.-M. Choi, Phys. Lett. B 513, 330 (2001); eConf C010430: T23 (2001).
 [5] B. L. G. Bakker, H.-M. Choi, and C.-R. Ji, Phys. Rev. D 63, 074014 (2001).
 [6] B.L.G. Bakker, H.-M. Choi, and C.-R. Ji, Phys. Rev. D 65, 116001 (2002).
 [7] D. Melikhov and S. Simula, Phys. Rev. D 65, 094043 (2002); S. Simula, Phys. Rev. C 66, 035201 (2002).
 [8] W. Jaus, Phys. Rev. D 67, 094010 (2003).
 [9] B.L.G. Bakker, H.-M. Choi, and C.-R. Ji, Phys. Rev. D 67, 113007 (2003).
 [10] S.-J. Chang and T.-M. Yan, Phys. Rev. D 7, 1147 (1973); Phys. Rev. D 7, 1780 (1973).
 [11] M. Burkardt, Nucl. Phys. A 504, 762 (1989).
 [12] J.P.B.C. de Melo, J.H.O. Sales, T. Frederico, and P.U.Sauer, Nucl. Phys. A 631, 574c (1998); J.P.B.C. de Melo, T. Frederico, H.W.L. Naus, and P.U. Sauer, Nucl. Phys. A 660, 219 (1999).
 [13] S. J. Brodsky and D. S. Hwang, Nucl. Phys. B 543, 239 (1998).
 [14] H.-M. Choi and C.-R. Ji, Phys. Rev. D 58, 071901 (1998).
 [15] N. C. J. Schoonderwoerd and B. L. G. Bakker, Phys. Rev. D 57, 4965 (1998); Phys. Rev. D 58, 025013 (1998).</p> | <p>[16] W. Jaus, Phys. Rev. D 41, 3394 (1990); 44, 2851 (1991).
 [17] W. Jaus, Phys. Rev. D 53, 1349 (1996); 54, 5904(E) (1996).
 [18] F. Cardarelli, I. L. Grach, I. M. Narodetskii, G. Salme, and S. Simula, Phys. Lett. B 349, 393 (1995).
 [19] D. Melikhov, Phys. Rev. D 53, 2460 (1996); Phys. Lett. B 380, 363 (1996).
 [20] H.-M. Choi and C.-R. Ji, Nucl. Phys. A 618, 291 (1997).
 [21] H.-M. Choi and C.-R. Ji, Phys. Rev. D 56, 6010 (1997).
 [22] H.-M. Choi and C.-R. Ji, Phys. Rev. D 59, 074015 (1999).
 [23] H.-M. Choi and C.-R. Ji, Phys. Rev. D 59, 034001 (1999); Phys. Lett. B 460, 461 (1999).
 [24] L.S. Kisslinger, H.-M. Choi, and C.-R. Ji, Phys. Rev. D 63, 113005 (2001).
 [25] H.-M. Choi, C.-R. Ji, and L.S. Kisslinger, Phys. Rev. D 65, 074032 (2002).
 [26] C.-W. Hwang, Phys. Lett. B 530, 93 (2002).
 [27] R. G. Arnold, C. E. Carlson and F. Gross, Phys. Rev. C 21, 1426 (1980).
 [28] I. L. Grach and L. A. Kondratyuk, Sov. J. Nucl. Phys. 39, 198 (1984).
 [29] P. L. Chung, F. Coester, B. D. Keister and W. N. Polyzou, Phys. Rev. C 37, 2000 (1988).
 [30] S. J. Brodsky and J. R. Hiller, Phys. Rev. D 46, 2141 (1992).
 [31] B. D. Keister, Phys. Rev. D 49, 1500 (1994).
 [32] J. D. Bjorken and S. D. Drell, <i>Relativistic Quantum Mechanics</i> (McGraw-Hill, New York, 1964); J. D. Bjorken</p> |
|---|---|

- and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).
- [33] J.P.B.C. de Melo and T. Frederico, Phys. Rev. C **55**, 2043 (1997).
 - [34] J.P.B.C. de Melo and T. Frederico, hep-ph/0311079.
 - [35] L. A. Kondratyuk and M. Strikman, Nucl. Phys. A **426**, 575 (1984).
 - [36] D. Arndt and C.-R. Ji, Phys. Rev. D **60**, 094020 (1999).
 - [37] T. Frederico and G. A. Miller, Phys. Rev. D **45**, 4207 (1992).
 - [38] R. Shtokhamer and P. Singer, Phys. Rev. D **7**, 790 (1973).
 - [39] T.M. Aliev, I. Kanik, and M. Savci, Phys. Rev. D **68**, 056002 (2003).
 - [40] A. Samsonov, J. High Energy Phys. bf 12, 061(2003).
 - [41] F. Hawes and M. Pichowsky, Phys. Rev. C **59**, 1743 (1999).
 - [42] M. Hecht and B.H.J. Mc Kellar, Phys. Rev. C **57**, 2638 (1998).
 - [43] Carl E. Carlson and C.-R. Ji, Phys. Rev. D **67**, 116002 (2003).
 - [44] Carl E. Carlson, J. Hiller, and R. Holt, Annu. Rev. Nucl. Part. Sci. **47**, 395(1997).
 - [45] B. L. G. Bakker and C.-R. Ji, Phys. Rev. D **65**, 073002 (2002).
 - [46] A. S. Bagdasaryan and S. V. Esaybegyan, Yad. Fiz. **42**, 440 (1985) [Sov. J. Nucl. Phys. **42**, 278 (1985)].